



Introduction



Aesthetics



Technicalities



Algorithm



Experimentation



Conclusion

Quantum Boundaries for Last Line Adjustments

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The Final Line

► word *glue* word *glue* word *glue* ... word $\longleftarrow \infty \text{ stretch} \longrightarrow$

Naturally overfull line

the fire-place, and a tall harpoon standing at the head of the bed.
the fire-place, and a tall harpoon standing at the head of the bed.

Naturally underfull line

and a tall harpoon standing at the head of the bed.

Problem... but not only!

faintly representing a tall straight jet of misty spray, and these words underneath – “The Spouter Inn: – Peter Coffin.”

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Aesthetic Considerations

Runt lines

be catastrophic, as I have artificially illustrated just here.

From a more abstract point of view, runt lines

Full-out lines

full justification was attempted but failed (again, as I have artificially demonstrated on this line's end).

💡 Two thresholds (runt 15% & full-out 5%) → two “avoid zones”

But the treatment is different

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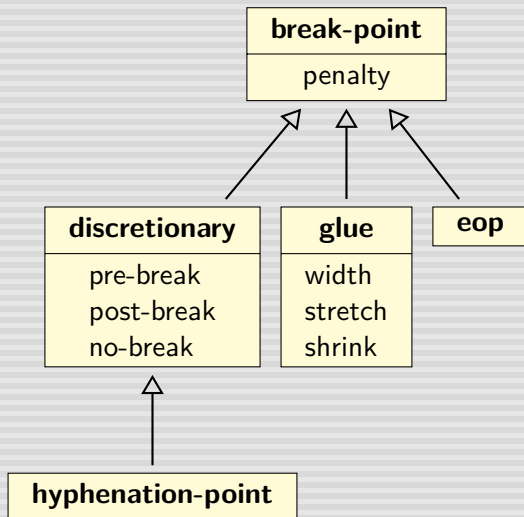
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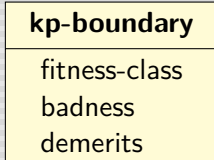
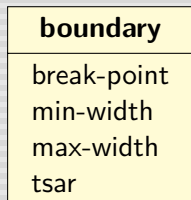
But the treatment is different

Problem Domain Data Structures



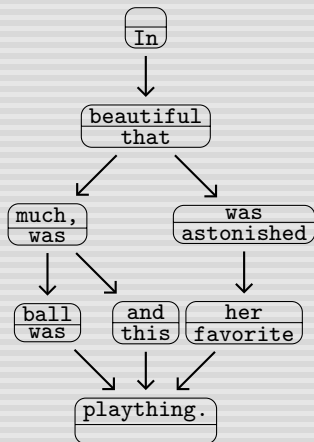
- ▶ penalty = property
- ▶ eop $\Rightarrow$ no final glue

Algorithmic Data Structures



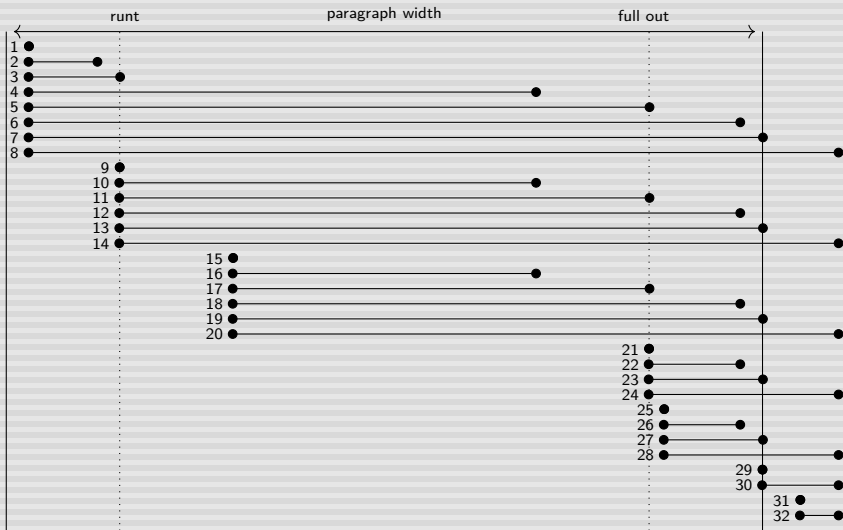
- ▶ min/max-width depend on tolerance
- ▶ TSAR = Theoretical Spacing Adjustment Ratio

Node Boundary Sharing

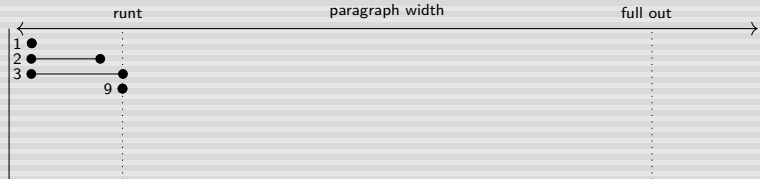


- ▶ Non-rectangular paragraphs
- ▶ $\neq$ fitness classes
- ▶ Final line adjustment!

Final Line Morphology



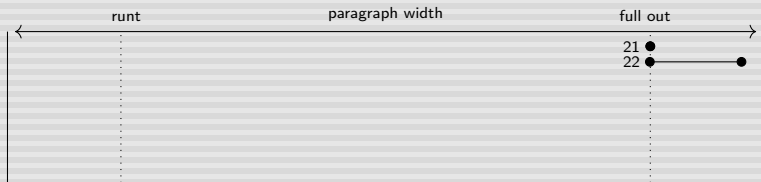
Deterministic Cases I



► Target width $\leftarrow$ runt

- 1, 2: underfull
- 3: adjusted
- 9: *nop*

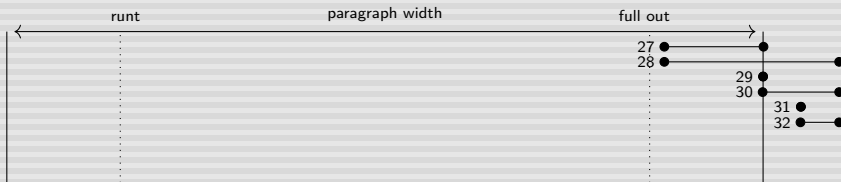
Deterministic Cases II



► Target width $\leftarrow$ full-out

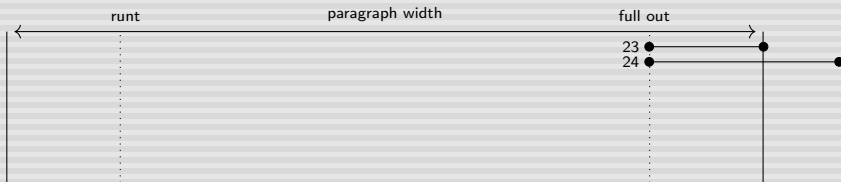
- 21: *nop*
- 22: adjusted

Deterministic Cases III



- ▶ Target width $\leftarrow$ paragraph width
 - ▶ 27, 28, 30: adjusted
 - ▶ 29: *nop*
 - ▶ 31, 32: overfull

Quantum Cases I



kp-boundary



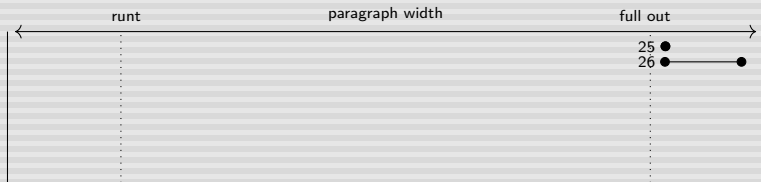
full-out-boundary

fsar
jsar

- ▶ 2 target widths
 - ▶ full-out → FSAR
 - ▶ paragraph width → JSAR

👉 $\text{TSAR} \leftarrow \min(|\text{FSAR}|, |\text{JSAR}|)$

Quantum Cases II



kp-boundary

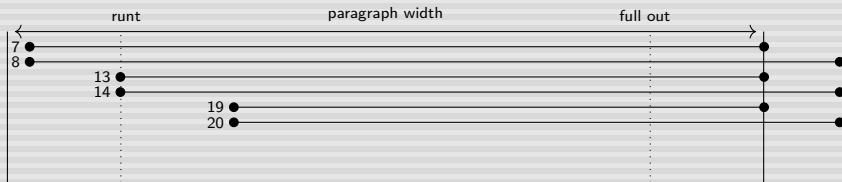


full-out-boundary

fsar
jsar

- ▶ 2 target widths
 - ▶ full-out → FSAR (overfull)
 - ▶ paragraph width → JSAR (underfull)
- 👉 TSAR ← JSAR

Quantum Cases III



kp-boundary



range-boundary

min-sar
max-sar
jsar

- ▶ A range of target widths + justification
 - ▶ $\text{min-sar} \leftarrow \max(\text{min-width}(\text{boundary}), \text{runt})$
 - ▶ $\text{max-sar} \leftarrow \min(\text{max-width}(\text{boundary}), \text{full-out})$
 - ▶ + JSAR for justification
- 👉 TSAR $\leftarrow$ minimize distance to natural spacing

Quantum Cases IV



kp-boundary

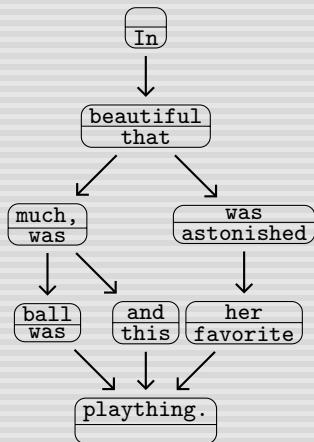


range-boundary

min-sar
max-sar
jsar

- ▶ A range of target widths / no justification
 - ▶ min-sar $\leftarrow \max(\text{min-width}(\text{boundary}), \text{runt})$
 - ▶ max-sar $\leftarrow \min(\text{max-width}(\text{boundary}), \text{full-out})$
 - ▶ JSAR $\leftarrow \text{nil}$
- 👉 TSAR $\leftarrow$ minimize distance to natural spacing

(Less) ~~Node~~ Boundary Sharing



- ▶ Non-rectangular paragraphs
- ▶ $\neq$ fitness classes
- ▶ Final line adjustment!
 - ▶ Deterministic cases
as usual
 - ▶ Quantum boundaries
cloned and adjusted at path creation time
- 👉 Minimize TSAR distance to previous line
- ⚠ Non-zero badness when stretching!

Experimentation

- ▶ 1279 paragraphs from Moby Dick @ 284 pt (≈ 10 cm)
- ▶ Latin Mordern Roman @ 10 pt
- ▶ Knuth-Plass with default settings +/- final line adjustments

Failures

- ▶ Original: 33 (2.5%)
- ▶ Additional: 4 (all intolerable runts)

Example

"Face!" said I, "call that his face? very benevolent countenance then; but how hard he breathes, he's heaving himself; get off Queequeg, you are heavy, it's grinding the face of the poor. Get off Queequeg! Look, he'll twitch you off soon. I wonder he don't wake." 

"Face!" said I, "call that his face? very benevolent countenance then; but how hard he breathes, he's heaving himself; get off Queequeg, you are heavy, it's grinding the face of the poor. Get off, Queequeg! Look, he'll twitch you off soon. I wonder he don't wake." 

Identical Solutions

- ▶ 1099 (86%)
- ▶ Acceptable lines: 978 (89%)

Example

Moving on, I at last came to a dim sort of light not far from the docks, and heard a forlorn creaking in the air; and looking up, saw a swinging sign over the door with a white painting upon it, faintly representing a tall straight jet of misty spray, and these words underneath – "The Spouter Inn: – Peter Coffin."

Moving on, I at last came to a dim sort of light not far from the docks, and heard a forlorn creaking in the air; and looking up, saw a swinging sign over the door with a white painting upon it, faintly representing a tall straight jet of misty spray, and these words underneath – "The Spouter Inn: – Peter Coffin."

Identical Solutions

- ▶ 1099 (86%)
- ▶ Adjustable runs: only 1 (by 0.66 pt)

Identical Solutions

- ▶ 1099 (86%)
- ▶ Adjustable full-outs: 70 (6%)

Example

There now is your insular city of the Manhattoes, belted round by wharves as Indian isles by coral reefs – commerce surrounds it with her surf. Right and left, the streets take you waterward. Its extreme downtown is the battery, where that noble mole is washed by waves, and cooled by breezes, which a few hours previous were out of sight of land. Look at the crowds of water-gazers there. ■■■

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Identical Solutions

- ▶ 1099 (86%)
- ▶ Adjustable overfulls: 50 (5%), all fully justified

Different Solutions

- ▶ 143 (11%): 119 same pass, 24 next pass
- ▶ Out of necessity: 108 (75%), all intolerable runts

Example

"Speak-e! tell-ee me who-ee be, or dam-me, I kill-e!" again growled the cannibal, while his horrid flourishings of the tomahawk scattered the hot tobacco ashes about me till I thought my linen would get on fire. But thank heaven, at that moment the landlord came into the room light in hand, and leaping from the bed I ran up to him. _____

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Different Solutions

- ▶ 143 (11%): 119 same pass, 24 next pass
- ▶ Out of choice: 35 (25%; 27 acceptable, 1 adjustable runt, 7 adjustable full-outs)

Example (⚠ non-zero badness when stretching!)

And since in this famous fishery, each mate or headsman, like a Gothic Knight of old, is always accompanied by his boat-steerer or harpooneer, who in certain conjunctures provides him with a fresh lance, when the former one has been badly twisted, or elbowed in the assault; and moreover, as there generally subsists between the two, a close intimacy and friendliness; it is therefore but meet, that in this place we set down who the Pequod's harpooneers were, and to what headsman each of them belonged.

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Additional Figures

- ▶ Pruning ratio: 7%
- ▶ Average lookup depth: 2.11
- ▶ Paragraph length: 20% — 33% shorter (by 1 line), otherwise equal

Conclusion

- ▶ Fine-tuning the final line is not so trivial
 1. Need to get rid of T_EX's original model
The final glue
 2. Not *only* a matter of spacing adjustment
Sometimes, need to look for a completely different paragraph breaking solution
- ▶ The algorithm can:
 1. adjust the spacing of the final line *w.r.t.* the previous one
 2. avoid runt and full-out lines
- ▶ Core idea: “quantum boundaries”
- ▶ Experimentation
 - ▶ 86% identical solutions
 - ▶ Runt lines are rarely adjustable
 - ▶ 25% of alternative solutions are due to stretching badness

Thank

you!